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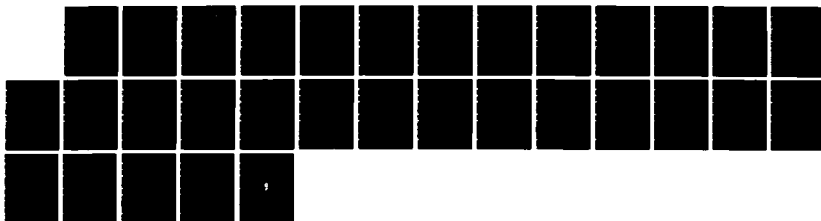
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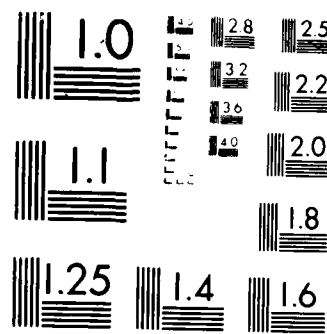
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**THEORETICAL INVESTIGATIONS OF  
RELEASES OF LITHIUM IN THE  
EARTH'S MAGNETOSPHERE**

FINAL REPORT

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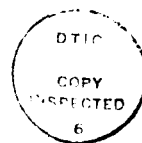
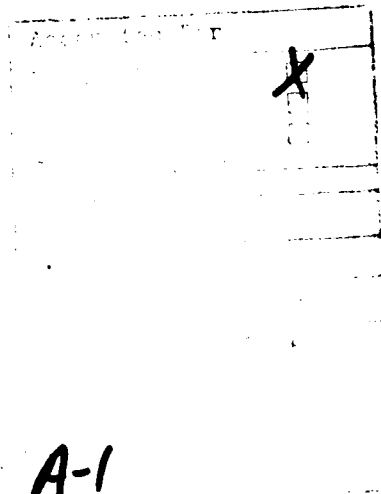
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# Theoretical Investigations of Releases of Lithium in the Earth's Magnetosphere

## Abstract

Physical processes determining the initial ionization of a chemical release cloud are considered. By analogy with the AMPTE lithium releases, it may be expected that the CRRES lithium releases will have a diamagnetic phase of a duration of the order of 30 seconds. The current distribution of the diamagnetic currents is determined, and implications for the generation of current-driven instabilities during the chemical release event are discussed.



# Theoretical Investigations of Releases of Lithium in the Earth's Magnetosphere

## I. Introduction

### I. A. General Discussion

Since the late 1950's there has been considerable interest in classes of experiments involving active modification of the plasma and magnetic field environment of near earth space. One of these experimental techniques involves the use of an energetic chemical reaction, (*i.e.* Ba with CuO) to release a large number,  $\sim 10^{25}$ , neutral metal atoms into space in a negligible time. These metal atoms are subsequently ionized by solar ultraviolet radiation, or in some instances by the Alfvén critical velocity effect (Haerendel 1982). As a significant collection of electrically charged particles, they constitute a perturbation of the magnetic field and plasma environment in some region of space.

This expanding plasma cloud will displace the ambient plasma and magnetic field, thus exhibiting a diamagnetic effect which is one of the principal concerns of this study. This implies the existence of significant surface currents in the release cloud and raises the possibility of current-driven plasma instabilities in the space surrounding the release cloud (see *e.g.* Gurnett *et al.* 1985), as well as the anomolous transport of particles by the waves and plasma turbulence generated by these processes (see *e.g.* Dum 1983). These and related effects may precipitate particles into the ionosphere, when the chemical release occurs in the earth's upper ionosphere or magnetosphere (Brice 1970), thus giving some insight to processes of energetic particle precipitation during geomagnetic storms. Eventually, the density of the chemical release cloud will become sufficiently low that it can no longer be considered as a coherent collection of plasma particles, and it will best be described in terms of a collection of single-particle trajectories in the geomagnetic

field. If such a chemical release is chemically distinctive, as is a lithium or barium release is in the geomagnetic environment, it will be useful as a "tracer" of the geomagnetic field configuration over some substantial region and can give significant information on the topology geomagnetic field lines and their connections to different regions of space (Chapman and Cowley 1984).

## I. B. Comparison of CRRES High Altitude Chemical Releases with Previous Chemical Release Experiments

The overwhelming majority of chemical release experiments performed to date have either been conducted at ionospheric altitudes, primarily due to the easy accessibility of this region with sounding rockets. There have also been a few releases from spacecraft in low earth orbit and some very well studied releases in the free-streaming solar wind and the magnetosheath in the AMPTE study. Most of these releases have involved easily ionized species such as barium, but a few have been releases of species with much larger ionization potentials, such as lithium, and also some electron-absorbing substances such as water.

The physics of chemical releases at low ionospheric altitudes is often dominated by the presence of a significant neutral background as well as a strong geomagnetic field. Velocities of the chemical release material with respect to the ambient medium have ranged from near zero (as in some sounding rocket releases) to values in excess of 10 kilometers per second (orbital velocity in low earth orbit of 8 kilometers per second, and higher velocities produced by shaped charges). The AMPTE releases involved a comparatively low ambient magnetic field, typical solar wind fields being  $\sim 5\gamma = 5$  nanoTesla, and the ram pressure due the streaming ambient plasma (velocities of hundreds of kilometers per second) the dominant source of pressure. In addition many interesting physical effects connected with polarization electric fields were noted (Haerendel, private communication).

The expected physical regime of the CRRES releases will be distinct from either of these previously well-studied cases. The strength of the geomagnetic field will be of an intermediate value,  $\sim 70 - 100\gamma$ . It is anticipated that the release will be made following a series of magnetic substorms, and therefore the magnitude of the ambient pressure will be comparable to that encountered in the AMPTE releases. However, a significant fraction



of that pressure will be due to nonthermal particles and the anisotropy of the pressure tensor must be considered. The velocity of the chemical release material with respect to the ambient plasma will be less than 1 kilometer per second, and therefore, it cannot be expected that polarization electric field effects (arising due to  $\mathbf{v} \times \mathbf{B}$ ) will be important.

Although the ionization time scales for lithium atoms exposed to solar ultraviolet are comparatively long ( $\tau \approx 2800$  sec.), a substantial diamagnetic phase lasting perhaps 30 seconds was noted in the AMPTE releases. This implies a substantial ionization during the early stages of the chemical release. Elementary calculations show that this cannot be due to any process arising in approximate thermodynamic equilibrium during the chemical release reaction. A possible explanation is that charged species produced as intermediate reaction products and in "side-reactions" are "quenched" in a thermodynamic nonequilibrium state by the rapid expansion of the cloud.

The existence of an initial diamagnetic phase, in addition to its intrinsic interest, will be of importance for a description of the state of the lithium release as it develops into a system of independent particles. Also, the short duration of the diamagnetic phase of the lithium release guarantees that the CRRES spacecraft will be in close proximity to the boundary of the cloud at the end of the diamagnetic phase, a time of particular theoretical interest.

## II. Physical Parameters of Magnetospheric Chemical Releases

### II. A. Charged Particle and Magnetic Field Environment for CRRES Releases

We shall be principally concerned with the CRRES lithium releases, which are anticipated to occur in the magnetosphere, just interior to the plasmapause, at  $L \approx 6$  at a time immediately following a magnetospheric substorm. As most well-studied chemical releases have occurred at ionospheric altitudes, where the plasma dynamics are dominated by the geomagnetic field, or in the magnetosheath or freely streaming solar wind (as in the AMPTE releases), where the magnetic field is very weak but the ram pressure of the convecting plasma is important, it is important to carefully consider the magnetic field and plasma environment in which the CRRES releases will occur.

The geomagnetic field, even at modest  $L$  values, can be significantly distorted from a dipolar configuration. These distortions arise due to the presence of the magnetopause, the geomagnetic tail, ring currents, and field-aligned currents (McPherron 1979, Lyons, 1979). At geosynchronous distances the field of the earth alone is approximately  $110\gamma$  while the perturbations may each contribute fields  $\sim 10 - 50\gamma$ , depending on the level of geomagnetic activity. The geomagnetic fields at the altitude planned for the CRRES release will be somewhat higher, and some of sources of perturbations will be somewhat less important, but these values are still indicative of the variability in magnetic field strength that may be anticipated. It is appropriate that we parametrize the value of the ambient magnetic field in these calculations so that we may cover the range of possible values which may be encountered during the CRRES releases. We shall choose a value of the unperturbed magnetic field of  $B_0 = 200\gamma$ , that is  $2.0 \times 10^{-3}$  gauss or 200 nanoTesla. This is taken to be the value of the magnetic field unperturbed by the chemical release, this does not represent a value of the magnetic field at  $L \approx 6$  for quiet times. The unperturbed vector magnetic field is then  $\mathbf{B}_0 = B_0 \hat{\mathbf{e}}_z$  where  $\hat{\mathbf{e}}_z$  is a unit vector in the  $z$  direction.

The distribution functions of electrons and protons (and other ions) will be very anisotropic, described by a pitch angle distribution. Accordingly, the pressure tensor will be anisotropic, with important implications for the expansion of the chemical release cloud. The energetic particles present here will be the dominant contributors to the pressure (not including for the moment the magnetic stresses of the distorted ambient magnetic field, which will also be anisotropic). This situation is to be compared with ionospheric chemical releases, where the presence of neutral atoms is important, or the AMPTE releases where the ram pressure of the solar wind was the most dynamically important pressure term. The magnitude of the pressure in the AMPTE chemical release and for the CRRES release, are however comparable in magnitude.

At the altitudes contemplated for the CRRES releases, after a magnetospheric substorm, energetic electron fluxes of the order of a few  $\times 10^7$  electrons/cm<sup>2</sup>·sr·sec in the energy range of 50 to 150 keV and fluxes  $\sim 10^6$  electrons/cm<sup>2</sup>·sr·sec in the energy range of 150 to 500 keV (Pfitzer and Winckler 1969). In addition to the importance of these comparatively energetic particles for their contribution to the plasma pressure, it is appropriate to consider their possible contributions to the ionization rate of a chemical release cloud.

## II. B. Plasma Diamagnetism in the Expanding Chemical Release Cloud

It can be expected that some analogy between the AMPTE and CRRES releases is possible, since the ram pressure of the solar wind in the earth's magnetosheath is comparable in magnitude to the total (magnetic field and thermal plasma) pressure in the plasmasphere following a large magnetospheric substorm.

Although the time scale for photoionization of lithium is much longer than that of barium, significant plasma diamagnetism was noted for the lithium releases in AMPTE. The duration of the diamagnetic phase in the AMPTE lithium releases was comparatively short, perhaps 10 seconds in all. This suggests that for CRRES all the important physics of the diamagnetic phase of the release will occur in close physical proximity to the spacecraft. This may have important consequences for using observations from CRRES for detailed tests of theories of plasma diamagnetism.

The dynamics of the AMPTE releases, particularly the barium releases, highlighted the importance of the polarization electric fields in understanding the physics of chemical releases. This was not well understood in theoretical treatments developed prior to the AMPTE releases. Comparatively simple analyses involving single particle trajectories in the polarization electric fields do much to explain the processes in the AMPTE releases. The role of polarization electric fields in magnetospheric lithium releases will require careful examination.

It was determined that the velocities of the lithium ions and neutrals generated by the chemical reactions in the AMPTE lithium release were low by approximately a factor of 0.7 in velocity or a factor of two in energy. It will be important for the CRRES lithium releases to understand the source of this discrepancy and to properly determine the appropriate initial conditions for the CRRES lithium release.

The basic outline of a theory of the diamagnetic phase of a chemical release can be given now. This outline will be filled out in greater mathematical detail and a computer simulation of the mathematical model will be constructed. During the time immediately following the release, gyrations of ions are unimportant compared to the cyclotron motions of electrons up until times of the order of the ion cyclotron period. As electrons have small mass and are easily accelerated, large currents may be obtained from comparatively small charge densities. This consideration is especially important for lithium releases since the ionization fraction is comparatively small, essentially the ionization produced during the chemical reactions of the release. Electron gyrations along with the perpendicular pressure gradient almost immediately generate a diamagnetic current (since the electron time scales are much less than ion time scales). Also, there exist contributions to the diamagnetic electron current arising due to the guiding center  $\mathbf{E} \times \mathbf{B}$  motion in the electric field. A radial space charge is created by these electron motions. This electric field couples to ion motions and drags the ions along with the electrons. The magnetic field inside the cavity is reduced by expansion of the diamagnetic cavity until pressure balance between the interior of the cavity and the total pressure of the ambient magnetized plasma is achieved. (Several bounce times of particles may be required for this pressure balance to be achieved.) Free expansion of plasma along the ambient magnetic field is limited only by magnetic stresses arising due to the distortion of the ambient magnetic field into a magnetic mirror-like configuration.

Information is being gathered characterizing in more detail the plasma and magnetic field environment of the plasmasphere at  $L \approx 6$  immediately following a magnetospheric substorm. These are the conditions anticipated for the CRRES releases.

## II. C. Physical Processes Determining the Ionization of the Chemical Release Cloud

Typically in chemical release calculations, the initial ionization of the released material is assumed to be zero. However, at these times early in the release, the initial temperatures and densities are sufficient to generate ionization fractions of  $10^{-4}$  or greater. These values of the ionization fraction will be "frozen-in" the initial distribution as the chemical release cloud rapidly expands so that the recombination rate falls off rapidly, and preserves a higher ionization fraction than would be expected on the basis of the instantaneous temperature. Furthermore, it is possible that side branches in the chemical reactions in the release may be a source of charged species as intermediate reaction products left behind as the reaction is "quenched" by the rapid expansion.

The relevant ionization mechanisms for modeling the ionization of the lithium release at early times have been identified and will be incorporated into the mathematical models of the diamagnetic phase of the growth of the chemical release cloud. Computer software has been designed and is being implemented for modeling the cloud growth and evolution of the magnetic field geometry during the diamagnetic phase of the chemical release.

Determination of the initial ionization state of the chemical release requires detailed consideration of the source and sink processes for electrons and ions under the conditions characteristic of the magnetosphere following a magnetic substorm, and of the relevant processes during the chemical release itself. Usually, a model is constructed which involves only photoionization of atoms ballistically streaming from a small initial volume with constant velocity,  $u_0$ , (Dum, 1983):

$$\frac{\delta n(\text{photo})}{\delta t} = \frac{\dot{N}_0(t - r/u_0)}{4\pi r^2 u_0 \tau_i} e^{-r/u_0 \tau_i} \quad (1)$$

Here  $n$  is the electron number density,  $\dot{N}_0$  is the total number of atoms (typically  $\sim 10^{25}$ ) released during the chemical release event,  $r$  is the distance from the release site, and  $\tau_i$

is the time constant for the ionization of atoms due to solar ultraviolet radiation. For barium,  $\tau_i$  is approximately 28 seconds, while for the lithium releases of primary interest for this research,  $\tau_i$  is approximately 3500 seconds.

A number of additional mechanisms require study in order to obtain an accurate description of the initial ionization state of the chemical release cloud. These include, but are limited to:

- 1) Initial production of charged species during the conditions of comparatively high temperature of the chemical release as well as charged species originating as radicals or intermediate reaction products which are "quenched" from the equilibrium state as the cloud expands.
- 2) Recombination processes which are of importance before the chemical release cloud density falls to very low values.
- 3) Charge exchange reactions with ambient charged species.
- 4) Ionization of neutral atoms in the chemical release cloud by energetic particles, *i.e.* radiation belt particles. It should be noted that the flux of these energetic particles is larger than typical of magnetospheric conditions during the interval shortly after a magnetospheric substorm.
- 5) Collisional excitation and de-excitation processes.

Some effort was spent in investigating the importance of each of these processes, with the view of either incorporating it in a theoretical description of the electrodynamics of the chemical release event, or demonstrating that it could be safely neglected.

We note here for later reference, that the ionization potential for singly-ionized lithium is  $I = 5.9$  eV. A characteristic temperature then is  $I$  divided by Boltzmann's constant,  $I/k = 6.26 \times 10^4$  K. This is significantly higher than the temperatures characteristic of the chemical release reactions,  $T \sim 5000$  K. Therefore, we cannot expect the initial lithium in the release to be substantially completely ionized, however, a small initial ionization

fraction, say  $f \sim 10^{-4}$ , would be significant for the diamagnetism of the cloud at early times.

Calculations were made concerning the collisional ionization equilibrium of a cloud of atomic lithium gas undergoing adiabatic expansion. It might be expected that if expansion is sufficiently rapid compared to time scales for the equilibration of the ionization fraction, that a significant level of ionization might be "frozen in" as the density of the cloud falls. However, it was found that detailed ionization equilibrium was maintained by a high collision frequency until the adiabatic expansion of the cloud had cooled the temperature sufficiently that ionization was completely negligible ( $f \ll 10^{-8}$ ).

A significant level of ionization may arise, however, from "side chains" from the main chemical reaction of the release, or as intermediate chemical species which are "quenched" as the density of the cloud drops as the cloud expands. At present there does not seem to be sufficient information available to reliably determine the contribution to the initial ionization state of the chemical release cloud due to these mechanisms.

Some theoretical attention was given to the problem of recombination of lithium ions and electrons as a competing process to their production by photoionization. Recombination coefficients of complex ions are dominated by recaptures to high quantum levels, which may to a good approximation be considered to be hydrogen-like (Osterbrock 1974). We may then find that the total recombination for singly-ionized lithium is  $\alpha \sim 2 \times 10^{-12} \text{ cm}^3/\text{sec}$ . A characteristic time for recombination is then  $\tau_r = 1/n_e\alpha$ , where  $n_e$  is the electron number density. This implies that  $n_e \sim 10^8 \text{ cm}^{-3}$  is required for recombination to be of any significance compared to photoionization. At these high densities, collisional ionization equilibrium is maintained over time scales characteristic of the expansion of the chemical release cloud. This implies that there is no physically significant regime for a chemical release event in the magnetosphere, in which the balance between photoionization and recombination is of physical importance.



Ionization of lithium atoms by radiation belt particles was also considered. At the altitudes contemplated for the CRRES releases, after a magnetospheric substorm energetic electron fluxes of a few  $\times 10^7$  electrons/cm<sup>2</sup> · sr · sec in the energy range of 50 to 150 keV and fluxes  $\sim 10^6$  electrons/cm<sup>2</sup> · sr · sec in the energy range of 150 to 500 keV (Pfützer and Winckler 1969). The cross sections for ionization by energetic electrons and for charge exchange reactions (for "energetic" protons, which have significantly smaller fluxes than the electron fluxes cited above) are of the order of  $10^{-16}$  cm<sup>2</sup> or less (Eletsky and Smirnov 1983). These numbers indicate characteristic times for ionization of  $10^8$  sec. or more, and so these processes are certainly of no significance compared to photoionization.

A number of processes of potential significance for the ionization state of a magnetospheric chemical release have been studied. With the possible exception of the production of charged species by chemical processes occurring during the release event, it has been demonstrated that only photoionization from solar ultraviolet is a significant source of ionization. Thus simple models of the ionization as given by Dum are appropriate, even for diamagnetism calculations of very early times in the chemical release. The improved understanding of the ionization within the chemical release developed by the calculations of the dynamics of the adiabatically expanding chemical release cloud, suggests strategies by which higher initial ionizations might be obtained.

### III. Diamagnetism of Chemical Release Clouds

#### III. A. Introduction - Qualitative Treatment Treatment of Diamagnetic Currents

In this section we are concerned with the determination of the currents in the boundary of the chemical release cloud which are physically responsible for the diamagnetism exhibited during the early stages of a chemical release. Since these currents are of possible importance for production of plasma instabilities and because it is desirable to carefully understand the decay of the diamagnetic phase of the chemical release, we shall give considerable attention to this problem.

It is appropriate to begin with a treatment of the quasi-stationary problem, *i.e.* in which the expansion velocity is not important. This is applicable as a first treatment of the problem as the expansion velocity of chemical release events is much less than the speed of light or of any characteristic plasma wave velocity. Therefore, the magnetic field configurations obtained may be treated as quasi-stationary until a magnetic diffusion time-scale is reached.

The objective is to determine a current distribution in the surface of the chemical release cloud which will yield a net magnetic field within the volume of zero. The magnetic field exterior to the volume will also be distorted from its initial uniform state by the currents in the diamagnetic volume. No exterior magnetic field will intersect the volume; the exterior magnetic field must be tangent to the surface of the chemical release cloud.

We shall approach the solution of this problem via a superposition technique. The problem is reformulated as the determination of the current distribution in a shell in empty space which yields a uniform magnetic field within the shell oppositely directed to the uniform background magnetic field in the final problem. The field produced is then

superposed on this uniform background, which produces a net magnetic field of zero within the volume and some distorted field exterior to the volume and close to the chemical release cloud.

Several avenues of attack on the determination of the current distribution are possible. Since the current distribution will be axisymmetric about the direction of the unperturbed magnetic field, it is natural to consider the current in the surface of the chemical release cloud as being composed of the sum of a collection of "infinitesimal" ring currents, at least for the purposes of numerical studies. The problem then is to determine a set of current values for rings distributed along the surface of the chemical release cloud which will approximately produce the desired field. Another approach which was very successful was to determine the vector potential of a current distribution in terms of a spherical harmonic expansion. The former treatment in terms of elementary rings, is therefore included for its theoretical interest and for the consideration of cases in which the chemical release cloud is very flattened.

### III. B. Diamagnetic Current Shell as a Superposition of Current-Carrying Rings

We shall be considering an axisymmetric current-carrying shell centered at the origin of a spherical coordinate system,  $(r, \theta, \phi)$ . The surface of the shell will be described by a curve  $a = a(\theta)$ . Each of the current rings will be centered on the  $\theta = 0$  line and parallel to the  $\theta = \pi/2$  plane. [Our treatment here follows (Jackson 1962).] Initially consider a single ring centered at the origin of the spherical coordinate, its current is given by

$$J_\phi = I \delta(\cos \theta') \frac{\delta(r' - a)}{a} \quad (2)$$

where  $\delta$  denotes the Dirac delta function, and  $I$  is the total current carried by the ring. The rectangular components of  $\mathbf{J}$  are required for the computation of the vector potential,  $J_x = -J_\phi \sin \phi'$ ,  $J_y = J_\phi \cos \phi'$ , and the vector potential is then determined by

$$\mathbf{A}(\mathbf{x}) = \frac{1}{c} \int \frac{\mathbf{J}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3 x' \quad (3)$$

Owing to the axisymmetry of the current distribution, we may choose our point of observation to lie in the  $xz$  plane, for purposes of this computation. Then the  $x$  component of the vector potential vanishes, leaving  $A_y = A_\phi$ , *i.e.*

$$A_\phi(r, \theta) = \frac{I}{ca} \int r'^2 dr' d\Omega' \frac{\cos \phi' \delta(\cos \theta') \delta(r' - a)}{|\mathbf{x} - \mathbf{x}'|} \quad (4)$$

where

$$|\mathbf{x} - \mathbf{x}'| = [r^2 + r'^2 - 2rr'(\cos \theta \cos \theta' + \sin \theta \sin \theta' \cos \phi')]^{1/2}.$$

Integrating over the delta functions, we may immediately obtain

$$A_\phi(r, \theta) = \frac{Ia}{c} \int_0^{2\pi} \frac{\cos \phi' d\phi'}{(a^2 + r^2 - 2ar \sin \theta \cos \phi')^{1/2}} \quad (5)$$

which can be expressed in terms of complete elliptic integrals of the first and second kind,  $K$  and  $E$ .

$$A_o(r, \theta) = \frac{4Ia}{c\sqrt{a^2 + r^2 + 2ar \sin \theta}} \left[ \frac{(2 - k^2)K(k) - 2E(k)}{k^2} \right] \quad (6)$$

where

$$k^2 = \frac{4ar \sin \theta}{a^2 + r^2 + 2ar \sin \theta}$$

The components of the magnetic field are then obtained from the definition,  $\mathbf{B} = \nabla \times \mathbf{A}$  expressed in spherical coordinates.

$$\begin{aligned} B_r &= \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_o) \\ B_\theta &= -\frac{1}{r} \frac{\partial}{\partial r} (r A_o) \\ B_o &= 0 \end{aligned} \quad (7)$$

Explicitly evaluating the components of  $\mathbf{B}$  in terms of derivatives of the vector potential as expressed in equation (6) is extremely tedious, even with recourse to symbolic manipulation routines, such as MACSYMA. A more practical approach used in the software contained in the appendix, is to numerically differentiate equation (6), to obtain the desired components of  $\mathbf{B}$ . The contributions to the magnetic field of a collection of rings may be determined by utilizing the formulae of this section with appropriate translation of the coordinates of the centers of the rings.

### III. C. Magnetization of a Current-Carrying Shell

We shall consider a current-carrying shell which is axisymmetric about the direction of the ambient magnetic field exterior to the shell. We want to determine a current distribution in the shell such that the magnetic field interior to the shell generated by the current is uniform and oppositely directed to the magnetic field exterior to the shell. The net effect, upon superposition of the magnetic fields exterior to the shell and that generated by the current distribution is to produce a zero net magnetic field within the current-carrying shell. This is the diamagnetic current distribution which we wish to determine. We may view the shell as possessing some uniform magnetization,  $\mathbf{M}$ .

We may calculate the vector potential produced by a given magnetization [Jackson 1962],

$$\mathbf{A}(\mathbf{x}) = - \int \nabla' \times \left( \frac{\mathbf{M}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \right) d^3x'. \quad (8)$$

In the case where the magnetization is due to a uniformly magnetized volume, the curl of the magnetization is nonzero only on the boundary of the volume, and we obtain,

$$\mathbf{A}(\mathbf{x}) = \oint \frac{\mathbf{M}(\mathbf{x}') \times \hat{\mathbf{n}}}{|\mathbf{x} - \mathbf{x}'|} da', \quad (9)$$

where  $da'$  is a surface element of the shell, and  $\hat{\mathbf{n}}$  is a unit normal, directed outward from the surface. The quantity,  $c(\mathbf{M} \times \hat{\mathbf{n}})$  represents a surface current density on the shell. Equation (9) is a general result applicable to an arbitrary volume bounded by an orientable surface. We shall be concerned with axisymmetric surfaces, since it is expected that the chemical release cloud will be axisymmetric about the direction of the geomagnetic field.

Begin by considering the case of a sphere, centered on a spherical coordinate system. Then

$$(\mathbf{M} \times \hat{\mathbf{n}})_\phi = M_0 \sin \theta'. \quad (10)$$

The magnetization and the magnetic field are related by  $B = \frac{8\pi}{3}M$ . Without loss of generality, we may choose observations (for purposes of evaluating the integral in (9)) to be made in the  $xz$  plane. We then have that

$$M_0 \sin \theta = \frac{4\pi}{c} K, \quad (11)$$

where  $K$  is the surface current density. Note that this surface current density is of the same form one would obtain by considering a sphere of radius  $a$ , with a charge  $Q$  uniformly distributed on its surface, and rotating with angular frequency  $\omega$ . Then the surface current density is given by

$$K = \frac{Q}{4\pi a^2} \omega (2\pi a) \sin \theta = \frac{\omega Q}{2a} \sin \theta$$

i.e.,

$$M_0 = \omega Q / 2a. \quad (12)$$

Since the observation point lies in the  $xz$  plane, only the  $y$  component of  $-(\hat{n} \times \mathbf{M})$  need be considered in the calculation of  $A_\phi$ .

$$A_\phi = M_0 a^2 \int d\Omega' \frac{\sin \theta' \cos \phi'}{|\mathbf{x} - \mathbf{x}'|} \quad (13)$$

Note that here  $\mathbf{x}'$  has spherical coordinates  $(a, \theta', \phi')$ . We may expand  $\sin \theta' \cos \phi'$  in terms of spherical harmonics,

$$\sin \theta' \cos \phi' = -\sqrt{\frac{8\pi}{3}} \Re[Y_{1,1}(\theta', \phi')]. \quad (14)$$

$\Re$  denotes the real part of the quantity. Now expand  $\frac{1}{|\mathbf{x} - \mathbf{x}'|}$  in spherical harmonics,

$$\frac{1}{|\mathbf{x} - \mathbf{x}'|} = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{1}{2\ell+1} \frac{r_-^\ell}{r_+^{\ell+1}} Y_{\ell m}^*(\theta', \phi') Y_{\ell m}(\theta, \phi). \quad (15)$$

Employing the orthogality property of spherical harmonics, we see that only the  $\ell = 1$ ,  $m = 1$  term survives in the integrand. Also,  $r_- = \min(r, a)$  and  $r_+ = \max(r, a)$ , and so we may evaluate the integral for  $A_\phi$ ,

$$A_\phi(\mathbf{x}) = \frac{4\pi}{3} M_0 a^2 (r_- / r_+^2) \sin \theta. \quad (16)$$

We may immediately employ equation (15) for calculating the magnetic field exterior to the spherical shell. For  $r < a$ ,

$$A_\phi = \frac{4\pi}{3} M_0 r \sin \theta, \quad (17a)$$

$$B_r(r < a) = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin^2 \theta \frac{4\pi}{3} M_0 r) = \frac{8\pi}{3} M_0 \cos \theta, \quad (17b)$$

and

$$B_\theta(r < a) = -\frac{1}{r} \frac{\partial}{\partial r} (\frac{4\pi}{3} M_0 r^2 \sin \theta) = -\frac{8\pi}{3} M_0 \sin \theta. \quad (17c)$$

Equations (17b) and (17c) describe a uniform magnetic field  $\mathbf{B} = B_0 \hat{z} = \frac{8\pi}{3} M_0 \hat{z}$ . For  $r > a$ ,

$$A_\phi(r > a) = \frac{4\pi}{3} M_0 \frac{a^3}{r^2} \sin \theta. \quad (18a)$$

$$B_r(r > a) = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\frac{4\pi}{3} M_0 \frac{a^3}{r^2} \sin^2 \theta) = \frac{8\pi}{3} M_0 \frac{a^3}{r^3} \cos \theta, \quad (18b)$$

$$B_\theta(r > a) = -\frac{1}{r} \frac{\partial}{\partial r} (\frac{4\pi}{3} M_0 \frac{a^3}{r} \sin \theta) = \frac{4\pi}{3} M_0 \frac{a^3}{r^3} \sin \theta. \quad (18c)$$

Clearly equations (18b) and (18c) describe a dipole magnetic field exterior to the spherical magnetized shell.

We can now consider the case of an arbitrary axisymmetric surface, which contains some surface current density which cancels out the effect of the ambient magnetic field within that volume. The surface is described by some curve  $a = a(\theta)$ . The condition that the surface current density is given by  $\frac{4\pi}{c} K \hat{e}_\phi = \mathbf{M} \times \hat{\mathbf{n}}$  is still valid (where  $\hat{e}_\phi$  is a unit vector in the  $\phi$  direction), but in general  $\hat{\mathbf{n}}$  will not be a radial vector, and so  $\mathbf{M} \times \hat{\mathbf{n}} \neq M_0 \sin \theta \hat{e}_\phi$ . The normal to the surface is given by

$$\hat{\mathbf{n}} = \frac{(\partial a / a \partial \theta) \hat{e}_\theta + \hat{e}_r}{\sqrt{1 + \frac{1}{a^2} (\partial a / \partial \theta)^2}}$$

and  $\mathbf{M} \times \hat{\mathbf{n}}$  may be computed using  $\mathbf{M} = M_0 \hat{e}_z$  and that we may evaluate quantities in the  $rz$  plane,

$$\begin{aligned} \mathbf{M} \times \hat{\mathbf{n}} &= \frac{M_0}{\sqrt{1 + (\partial a / a \partial \theta)^2}} \left\{ \frac{\partial a}{a \partial \theta} [\hat{e}_z \times (\hat{e}_x \cos \theta - \hat{e}_y \sin \theta)] + \hat{e}_z \times (\hat{e}_x \sin \theta + \hat{e}_y \cos \theta) \right\} \\ &= \frac{M_0}{\sqrt{1 + (\partial a / a \partial \theta)^2}} \left[ \frac{\partial a}{a \partial \theta} \cos \theta + \sin \theta \right] \hat{e}_y. \end{aligned} \quad (19)$$



Converting back to spherical coordinates,

$$\mathbf{M} \times \hat{\mathbf{n}} = \frac{M_0}{\sqrt{1 + (\partial a / a \partial \theta)^2}} \left[ \frac{\partial a}{a \partial \theta} \cos \theta + \sin \theta \right] \hat{\mathbf{e}}_\phi. \quad (20)$$

It is apparent that this formula exhibits the correct limiting behavior when  $a = \text{constant}$ .

The vector potential may now be computed from

$$A_\phi(\mathbf{x}) = M_0 a^2 \int d\Omega' \frac{\cos \phi'}{\sqrt{1 + (\partial a / a \partial \theta)^2}} \frac{(\sin \theta' + \frac{1}{a} \frac{\partial a}{\partial \theta'} \cos \theta')}{|\mathbf{x} - \mathbf{x}'|}. \quad (21)$$

As in the case of a spherical shell, we shall want to expand the integrand,

$$g(\theta', \phi') = \frac{\cos \phi' (\sin \theta' + \frac{1}{a} \frac{\partial a}{\partial \theta'} \cos \theta')}{\sqrt{1 + (\partial a / a \partial \theta)^2}},$$

in spherical harmonics. As before, the only  $\phi'$  dependence is contained in the  $\cos \phi'$  term, and so only the  $m = 1$  term survives the integration. In general, the coefficients of the spherical harmonic expansion will have to be determined numerically unless  $a(\theta)$  has some unusually simple functional form.

### III. D. Construction of a Dynamic Model of a Chemical Release Cloud during the Diamagnetic Phase

The calculations of the previous sections pertaining to the electrodynamics of a chemical release cloud during the diamagnetic phase of the expansion may be converted into a dynamical model of the expansion suitable for solution by numerical techniques.

The pressure within the chemical release cloud may be assumed to be initially uniform and the cloud nearly spherical, if a sufficiently early time is adopted for the modeling. At this time the configuration of the ambient magnetic field is found by solving the magnetostatic boundary value problem using the spherical harmonic techniques developed in previous sections. The use of magnetostatic equations is appropriate for this problem since the expansion velocity of the chemical release cloud is much slower than the local Alfvén velocity.

Once the configuration of the magnetic fields is known, the Maxwell stress tensor may be computed. This gives the effect of the magnetic stresses in opposing the expansion of the chemical release cloud. The anisotropic pressure of the magnetic field may be combined with the anisotropic pressure of the particle distribution (*e.g.* a loss-cone distribution). This then represents the total anisotropic pressure tensor acting on the chemical release cloud. This pressure tensor may be used with the cloud normal to determine the net pressure acting on a surface element of the chemical release cloud.

This force may then be compared with the gas pressure of the cloud, and the dynamic pressure (ram pressure) connected with the expansion of the cloud to compute the net acceleration of a surface element of the cloud. This must be done as a function of  $\theta$  to take into account the possibility of an anisotropic expansion of the cloud. The acceleration of the cloud may be integrated in time to obtain a new set of locations of surface elements of the cloud. The configuration of the external magnetic fields is then re-evaluated in

the magnetostatic boundary value problem and the calculation proceeds further in this fashion.

Ultimately the resistivity of the cloud permits diffusion of the magnetic field into the cloud. This effect is taken into account by computing the resistive energy loss in each time step, determined by the local plasma parameters and the surface current in the diamagnetic plasma cloud, which permits the inward diffusion of magnetic field.

### III. E. General Considerations on the Triggering of Plasma Instabilities by Diamagnetic Currents

One of the motivations for the study of diamagnetic currents in chemical releases is their possible role in the generation of current-driven plasma instabilities. The calculations of the previous sections have treated the diamagnetic currents as infinitesimally thin current sheets at the boundary of the chemical release cloud. In order to correctly assess the possibility of generation of current-driven instabilities, it will be necessary to obtain a more physically reasonable measure of the actual current densities associated with the chemical release event. It may be expected that the diamagnetic current will initially (early in the diamagnetic phase) be carried in a layer with a thickness of the order of the electron inertial length,  $c/\omega_{pe}$ , where  $\omega_{pe}$  is the electron plasma frequency which must be evaluated for the conditions at the immediate interior of the boundary of the shell. The surface current densities calculated in the previous two sections, together with this characteristic thickness will suffice to obtain a reasonable estimate for the current densities early in the chemical release. Later in the diamagnetic phase, it may be expected that the thickness of the current-carrying layer will increase as magnetic field begins to diffuse into the chemical release cloud. Therefore, the current densities will decrease for this reason, as well as their decay due to resistive losses.

The generation of current-driven instabilities which may be detected by spacecraft instruments in space exterior to the chemical release cloud requires that at least some of the magnetic field lines from the exterior plasma penetrate the chemical release cloud. (This pertains to plasma instabilities generated directly by the diamagnetic currents. There will also be currents flowing in the exterior plasma, as evidenced by the curl of the exterior magnetic field.) Accordingly, detailed study of the magnetic field geometries and current densities is required through the end of the decay of the diamagnetic phase of the

chemical release to fully understand the potential for generation of current-driven plasma instabilities.

### III. F. Comparison with the Results of the Tensor Virial Method

The tensor virial method is an alternative approach toward treating the dynamics of a cloud embedded in a magnetized plasma. This approach has been studied by Parker [1957] and other workers.

Through the construction of appropriate virial equations, the magnetic stresses and pressure distribution may be used to compute the second time derivative of the moments of inertia of the cloud. Unfortunately, this method is analytically intractable except for a few special cases, notably clouds which are internally homogeneous and whose boundaries are ellipsoidal. Given these conditions, Parker has shown that the expansion of the cloud is described by a sequence of ellipsoidal shapes which typically become more prolate with time. (Evolution toward more oblate ellipsoids occurs only when a tangled magnetic field interior to the cloud is important.)

Although the tensor virial method is too analytically complex to allow treatment of a general sort of expanding cloud, as might be produced by a chemical release experiment, we can gain some useful results which will serve to guide numerical modeling. Notably, the tendency of a cloud with no internal magnetic field expanding against a medium with a strong magnetic field to become more prolate is shown in some detail. This may be expected to be an important feature of the CRRES releases if the diamagnetic phase of the chemical release is sufficiently prolonged. Furthermore, this formalism may be of some utility in considerations of the process by which low altitude (*i.e.* ionospheric) chemical releases are broken up into filamentary structures. Detailed numerical analyses of the diamagnetic phase of chemical releases will be required to further elucidate these questions.

#### IV. Conclusions

The physical processes giving rise to diamagnetic currents early in a chemical release are considered. Even for releases of lithium under magnetospheric conditions, as contemplated for CRRES, it may be expected that there will be a diamagnetic phase of duration of approximately 30 seconds. This diamagnetic phase can be well studied owing to the close proximity of CRRES to the regions in which physically relevant processes will occur.

Mathematical formalisms were developed for detailed treatment of the diamagnetic phase of the evolution of a chemical release cloud. Further work will involve the decay of the diamagnetic phase and treatment of the generation of plasma instabilities as a result of the chemical release.

## V. References

- Brice, N. M. (1970), "Artificial Enhancement of Energetic Particle Precipitation through Cold Plasma Injection: A Technique for Seeding Substorms?", *J. Geophys. Res.*, **75**, 25
- Chapman, S. C., and Cowley S. W. H. (1984), "Acceleration of Lithium Test Ions in the Quiet Time Geomagnetic Tail", *J. Geophys. Res.*
- Dum, C. T., (1983), "AMPTE: Diamagnetic Effect and Anomalous Transport in Expanding Plasma Clouds", M.P.I. Report, ISSN 0340-8922
- Eletsky, A. V., and Smirnov, B. M., (1983), in **Basic Plasma Physics I**, (eds. Galeev, A. A., and Sudan, R. N.), North-Holland Publishing Co., New York, p. 61 ff
- Gurnett, D. A., *et.al.* (1985), "Plasma Waves Associated with the AMPTE Artificial Comet", *Geophys. Res. Lett.*, **12**, 851
- Haerendel, G. (1982) "Alfvén's Critical Velocity Effect Tested in Space", *Z. Naturforsch.*, **37a**, 728
- Jackson, J. D. (1962), **Classical Electrodynamics**, John Wiley & Sons, Inc., New York
- Lyons, Lawrence L. (1979), "Magnetic Field Distortions at  $L = 3-4$  Inferred from Simultaneous Low and High Altitude Observations of Ring Current Ions", in **Quantitative Modeling of Magnetospheric Processes**, Olson, ed., American Geophysical Union, Washington, D. C.
- McPherron, Robert L. (1979), "The Synchronous Orbit Magnetic Field Data Set", in **Quantitative Modeling of Magnetospheric Processes**, Olson, ed., American Geophysical Union, Washington, D. C.
- Osterbrock, D., (1974), **Astrophysics of Gaseous Nebulae**, W. H. Freeman & Co., San Francisco, p. 33 ff
- Parker, E. N. (1957), "The Gross Dynamics of a Hydromagnetic Gas Cloud", *Ap. J. Suppl.*, **3**, 51
- Pfitzer, K. A., and Winckler, J. R. (1969), *J. Geophys. Res.*, **74**, 5005



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